Hidden Volatility and Liquidity Risk Factors*

Dario Brandolini (*RAS Asset Management*)
Giampaolo Gabbi (*Università di Siena; SDA Bocconi*)
Claudio Marsala (*RAS Asset Management*)
Massimiliano Pallotta (*RAS Asset Management*)
Raffaele Zenti (*RAS Asset Management*)

Abstract

The main purpose of the paper is to define an algorithm of liquidity risk for bonds, since very frequently their volatility price is lower than what appears after some shocks. The expected output will be generated comparing qualitative and quantitative methodologies, such as bond portfolio managers and statistical analysis.
The explaining variables for bond liquidity risk are: currency, exchange, issue date, maturity, coupon type, coupon, duration, yield, rating Moody, rating S&P, defaulted and outstanding. Moreover, the existence of some options.
The main factors which lead to the estimation of liquidity risk are the outstanding of the issue and the default of the bond.
The outcome of the algorithm will be compared to the qualitative indications of some senior portfolio managers so to calibrate the model.

* We acknowledge the contribution of Bastianina Salis for the elaboration of data. The usual disclaimer applies.

1. Existing literature analysis

The state-of-the-art on liquidity risk volatility can be organised into three different categories:

1) Models where liquidity risk depends on expected execution lag for the execution of orders for securities (execution lag models);
2) Models where liquidity risk is function of *bid-ask spread* or implicit measures of the same spread (bid-ask spread models);
3) Models where the liquidity risk is measured through the outstanding traded in the market (volume models)

Execution lag models

Jarrow and Subramanian [1997] consider the selling lag and the price elasticity of a portfolio of bonds (or a part of it, c[L]) that the owner wants (or must) liquidate in the market [1].

$$\mathrm{VAR} = n\left\{\sigma E[\Delta(L)]^{1/2} + \left|\left(\mu - \frac{\sigma^2}{2}\right)\right| \cdot \sigma[\Delta(L)] + \sigma[\log\ c(L)]\right\} \qquad [1]$$

where:

VAR is the value-at-risk, that is "the largest likely loss from market risks (expressed in currency units) that an asset or portfolio will suffer over a time interval and with a degree of certainty selected by the decision-maker" [Titus - Lewis, 1997; for an overview see Barone-Adesi, Giannopoulos, Vosper, 2001];

n depends on the underlying distribution;

$E[\Delta(L)]$ is the expected execution lag in selling the L shares;

μ is the mean of the quantity discount;

σ is the volatility of the quantity discount;

c(L) is the quantity discount.

Practical limit is data availability to estimate worst case scenarios. To build the Jarrow-Subramanian model we need database for:

1) price discount;
2) price discount volatility;
3) time liquidity volatility.

Computation of liquidity risk adjusted VaR according to this model depends upon the knowledge of the following factors:

a) mean and variance of interbank funding rates;

b) mean and variance of time needed to borrow money aimed at gross settlement[1];
c) mean and variance of interest rates elasticity to time.

If variables a) and b) can be estimated, information sub c) lack of a reliable statistical providing.

A potential alternative model is to estimate a bond demand function depending upon both market observable variables and a stochastic factor. This model, originally proposed by Madhavan e Smidt (1991), explains the orders volume q_t according to the equation [2]

$$q_t = \text{á}(m_t - p_t) + \text{÷}_t \qquad [2]$$

where:
m is the bond fair value;
p is the market price;
÷ is the random variable explaining the sum of the orders not depending on the available information but on liquidity.
In this case, the selling cost (cs) is:

$$cs = 2(\text{ø} / \text{ð} + \text{ë}\left|q_t\right|) \qquad [3]$$

where
Ø is the cost of the single trade;
ð is a coefficient between 0 and 1;
ë is the price effect for the single trade.
Unfortunately, even this model needs a database particularly complicated to build.

Bid-ask spread models

In order to ease the database problem, Cherubini [1997] models the liquidity risk within the model of [1] by the proxy of bid-ask spread in the market [4].

[1] The average settlement delay observed in BIREL for large value payments is very low—less than two minutes—due to the structure of the system (centralised queuing mechanism and unlimited supply of intra-day liquidity through fully collateralised overdrafts). In fact, the screen-based self-regulated Italian interbank market does not quote intra-day liquidity funds. Moreover, according to the main Italian banks, the time-criticality of payments settled in the RTGS system seems to be very low and, up to now, the settlement delays are not perceived as costly by banks and customers, Impenna and Masi [1998].

$$\mathrm{VAR} = n\sigma\sqrt{\sigma^2[\Delta_{\mathrm{BID-ASK}}] + \sigma^2_{\mathrm{MKT}} + \rho\sigma\ [\Delta_{\mathrm{BID-ASK}}]\cdot\sigma_{\mathrm{MKT}}} \qquad [4]$$

Other papers based on bid-ask spread are proposed by Schulz [1998], Chakravarty – Sarkar [1999] Almgren – Chriss [1999], Hong-Warga [2000] and Gwilym-Trevino-Thomas [2002]

This approach is coherent with the examination procedures listed by the Federal Reserve [1998]: points 5.b) and 5.c) ask to obtain all management information, reviewing bid/ask assumptions in a normal market scenario and reviewing stress tests that analyse the widening of bid/ask spreads and determining the reasonableness of assumptions.

Bangia et al. [1999] verify the historical behaviour (back-testing) of this model for portfolios characterised by remarkable market shocks: in case of liquidity risk adjusted VaR is shown as the violations are statistically acceptable (green zone indicated by the Bank for International Settlements): this means that banks able to implement liquidity risk in VaR computations could lower the weighted average cost of capital.

The exogenous cost of liquidity (COL) they find is based on a certain average spread, $\overline{S}$, plus a multiple of the spread volatility, $a \cdot s_{spread}$:

$$COL = \frac{1}{2}\left[P_t\left(\overline{S} + as_{spread}\right)\right] \qquad [5]$$

where P_t is today's mid price for the asset or instrument, $\overline{S}$ is the average relative spread, defined as [Bid–Ask]/Mid price, i.e. a normalising device which allows for easy comparison across different instrument, s_{spread} is the volatility of relative spread, a is the scaling factor.

In order to calculate the liquidity risk adjusted VaR they incorporate the 99th percentile movement in the spread, jointly with 99th percentile movement in the underlying.

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This model hypotheses a complete variability of interbank interest rates and bid-ask spreads, but in interbank markets we can find some constraints. In fact, usually, monetary policy authorities fix a corridor made by the official deposit rate and the marginal lending rate. These conditions apply for banks with collateral.

A potential problem for this model is the significance of the bid-ask information (when available). In this case it could be possible to introduce the slippage, defined as the difference between the execution average price and the average provided bid-ask spread. This way, we can estimate the different prices the market can absorb, to measure the average price with respect the ideal price originally stated by the market makers.

This kind of data not necessarily is offered by the primary information providers.

A different solution is to estimate some indicators able to proxy the selling cost: Roll [1984] models the spread with the equation [6]:

$$\mathbf{Spread} = 2 \bullet \sqrt{-\operatorname{cov}(\Delta p_t, \Delta p_{t-1})} \qquad [6]$$

Through the equation [6] is possible estimate the spread very easily: let us suppose that the standard deviation of price spreads is 25 basis points. Then we estimate an autoregressive model of first order as follows:

$$\Delta p_t = 0{,}00001 - 0{,}0064 \Delta p_{t-1} \qquad [7]$$

This means that $-\operatorname{cov}(\Delta p_t, \Delta p_{t-1}) = 0{,}0064 \cdot\ 0{,}00000625$. The Roll spread equals 0,0004 (4 basis points), which stands for 400 euro of selling cost for every bond position of million euro.

Volume models

The third category of liquidity risk models is based on a set of proxies depending on volume or outstanding of the bonds.
Pàstor and Stambaugh model the bond yield as follows:

$$r^e_{i,d+1} = \alpha + \beta \bullet r_{i,d} + \chi \bullet \operatorname{sign}(r^e_{i,d}) \bullet vol_{i,d} + \varepsilon_{i,d+1} \qquad [8]$$

where:
$r_{i,d}$ is the yield of bond i in day d;
$r^e_{i,d+1}$ is the difference between bond yield and the market's in the same day;
$vol_{i,d}$ is the volume of transactions traded over the bond during the day d;
The hypothesis is that for illiquid bonds, the yield should be partially opposite to the market one.

Other papers based on volume are by Kamara [1994] and Alexander, Edwards and Ferri [2000].

All these models depend on time series database, which are, usually complicated to obtain from the information providers. To the best of our knowledge, no study has empirically examined the relevance of different “static” factors in determining corporate bonds’ liquidity. This way, risk managers could identify bond liquidity in all the time of their life, even in the primary market, when there is lack of time series information.

2. Data

The main purpose of our study is to recognize liquidity or illiquidity analyzing some key variables known in a static way and not by studying the price time series.

The explaining variables, that we believe a priori related to the concept of liquidity for bond:

1. currency;
2. exchange;
3. date time;
4. maturity;
5. coupon type;
6. credit spread;
7. coupon;
8. duration;
9. yield;
10. rating by Moody;
11. rating by S&P;
12. issuer rating;
13. if defaulted or not;
14. outstanding;
15. options;

We built a database for 174 international bonds. The issuers are both sovereigns (69) and corporates (103); they are denominated in U.S. dollars (38), Canadian dollars (3), euros (110), pound sterling (11), and Swiss francs (10); both investment-grade and speculative ones are included; bonds are listed in primary exchange, such as Euronext (9), Frankfurt (11), Milan (1), London (15), New York (36), Luxembourg (72), EBS Swiss (9), Madrid (3), Stockholm (2) and, in some cases, not listed (14). Some bonds are defaulted (6). Considering the issue date, the time interval is from 14 years to two months.

Moreover, the existence of some option such as bearer, Brady, bullet, callable, convertible, credit linked, exchangeable, index linked, make whole call, private placement, putable, registered, reverse convertible, sinkable, soft call, structured note will be considered.

We expect that some of these variables are more relevant to cause illiquidity: in particular, if a bond is either defaulted, or not listed this is a good warning of its low price reliability.

Moreover, some other illiquidity expectations are currencies out of the primary basket, defaulted coupons, high decay time, the absence of the credit spread and the rating in the information providers[2]. Finally, when bonds are particularly structured with options difficult to understand it is easier to believe they are less liquid than vanilla bonds.

On balance, our variables are twelve, either directly evaluated or transformed in dummies (Table 1).

Table 1 – Typology of variables

Name of variable	Not dummy	Dummy
V1	Numeration of bonds	-
V2	Defaulted bond	If defaulted or not
V3	Name of exchange	Listed or not listed (out of the market)
V4	Outstanding	Higher of lower than 500.000 euros
V5	Currency	Primary or residual currency
V6	Coupon type	Fixed or not
V7	Time period from issue date	Lower or higher than 1 year
V8	Maturity	Lower or higher than 1 year
V9	Credit spread	If existing or not
V10	Bond Rating	If rated or not
V11	Issuer Rating	If rated or not
V12	Number of options	Lower or higher than 1

Since statistical analysis depends on distribution of explicative variables, we define skewness and kurtosis (Table 2) which appear to be far from being normal.

[2] We do not consider the level of bond rating to avoid any credit risk consideration.

Table 2 – Normality statistics

Name of variable	Skewness	Kurtosis
V2	-5,15	24,77
V3	-3,62	11,24
V4	-0,15	-2,01
V5	-2,05	2,23
V6	-5,15	24,78
V7	-0,66	-1,59
V8	-0,92	-1,17
V9	1,04	-0,92
V10	-1,18	-0,61
V11	0,8	-1,38
V12	-0,33	-1,91

3. Methodology

The study follows two steps:

1) firstly, we asked two different bond portfolio managers to indicate if liquid or not all the securities of our database;

2) secondly, though two statistical methodologies (the discriminant and the logit analysis), we tried to estimate the theoretical equations able to generate the same empirical outcomes.

Since to perform the discriminant and the logit the groups of data should be equally shared by the selective variable (in this case liquidity and illiquidity), we obtain a couple of datasets uniformly judged by our experts.

Since the two experts (A and B) did not answer in the same way, we have two groups of datasets. A third one (C) has been obtained as the merge of the previous ones when both A and B agree to each other.

The group A is composed by 83 illiquid and 83 liquid bonds. The group B is composed by 54 illiquid and 54 liquid bonds. Finally, the merging group dimension is of 96 bonds, equally shared in liquid and illiquid.

The discriminant analysis has been applied in four ways:

1. without mean: when there is a missing value the bond is erased from the dataset [WM];

2. with mean, which means that when there are missing values the dataset replaces with the mean [M];
3. stepwise without mean [SWM];
4. stepwise with mean [SM].

The group of variables have been elaborated in three ways:

1. without dummies: we apply the dataset with the exact variables [WD];
2. with dummies: we substitute all the variables with dummies (see Table 1) [D];
3. with dummies equally distributed [DL].

This means that we expect to estimate twelve different discriminant models for each portfolio manager and for their join dataset (Table 3).

Table 3 – Format of discriminant analysis results

		GROUP OF VARIABLES		
		WD	D	DL
DISCRIMINANT ANALYSIS MODELS	WM M SWM SM			

The logit analysis does not present the four models, thus we will estimate only three solutions for each dataset (Table 4).

Table 4 – Format of logit analysis results

		GROUP OF VARIABLES		
		WD	D	DL
GROUP OF DATASETS	A B C			

4. Empirical results

Our statistical results will be compared with the expected output, which depends on the opinion of portfolio managers we interviewed. Criteria to choose among different models are robustness and minimization of errors of I (considering liquid an illiquid bond) and II type (considering illiquid a liquid bond).

Discriminant model

The first group of outcomes we present is generated by the application of a very well known statistical model, already applied in financial risk management studies [Altman, 1977; Altman-Varetto, 1993; Falbo, 1991; Gordon-Palmer-Glover, 1993] especially in order to investigate the credit risk factors.

While in the quoted articles, the classification of events is objective (the firm is either sound or bankrupt), in our case it depends on the subjective experience of portfolio managers.

Since this difference, to model the liquidity risk to get it more fitted to the phenomenon we probably need more variables, than those generally used in credit risk literature.

Table 5 – Discriminant analysis results for portfolio manager "A"

PORTFOLIO MANAGER "A"		GROUP OF VARIABLES		
		WD	D	DL
DISCRIMINANT ANALYSIS MODELS	WM	65,7	68,4	85,1
	M	88,5	66,7	83,6
	SWM	-	70,9	81,8
	SM	-	68,6	81,3

* The average expected value is 50,0%

Table 6 – Discriminant analysis results for portfolio manager "B"

PORTFOLIO MANAGER "B"		GROUP OF VARIABLES		
		WD	D	DL
DISCRIMINANT ANALYSIS MODELS	WM	92,6	90,2	96,0
	M	78,8	81,7	89,6
	SWM	84,6	87,8	96,0
	SM	84,6	77,9	89,0

* The average expected value is 50,0%

Table 7 – Discriminant analysis results for merge group “C”

MERGE GROUP “C”		GROUP OF VARIABLES		
		WD	**D**	**DL**
DISCRIMINANT ANALYSIS MODELS	**WM**	100,0	94,3	**96,7**
	M	72,5	86,4	94,1
	SWM	82,6	88,7	96,7
	SM	82,6	78,8	94,1

* The average expected value is 50,0%

Statistical robustness suggests to choose not the best model (WD-WM) but the second one (DL-WM). The reasons are:

a) the perfect result we obtain depends on few data which are elaborated in the WD-WM model;
b) all the other WD models show a lower capability to minimize the I and II type of errors.

Table 8 shows the percentage of good and bad results of error of the (DL-WM) for group C, which merges the experience of two portfolio managers.

Table 8 – Outcome Classification (DL-WM) for group C

		Forecast	
		Illiquid	Liquid
Actual	Illiquid	97,67%	2,33%
	Liquid	3,75%	96,25%

Once verified the accuracy of the model, we present the coefficients and the significance tests (Table 9) which appear to be very high.

Table 9 – Equation (DL-WM) for group C

Variables	Coefficients	F-test	Significance
Constant	-3,958	6,378	0,000
V2	0,404	10,355	0,002
V3	0,294	17,988	0,000
V4	5,419	772,558	0,000
V5	0,330	7,529	0,007
V6	0,364	8,072	0,005
V7	0,114	22,213	0,000
V8	-0,591	12,350	0,001
V9	-0,693	14,148	0,000
V10	0,889	13,067	0,000
V11	-0,685	16,151	0,000
V12	-1,294	82,549	0,000

Logit model

The fitness of the logit models is presented in Table 10. Since, even in this cases, the expected value is 50 per cent, all our outcomes show a good appropriateness.

Table 10 – Logit analysis results (percentage values)

		GROUP OF VARIABLES		
		WD	**D**	**DL**
GROUP OF DATASETS	**A**	61,5	69,6	85,1
	B	74,1	95,1	96,0
	C	-	98,1	**98,4**

* The average expected value is 50,0%

As expected, the best models are those generated from DL group of variables.

Analysing the I and II type of errors of DL models, we obtain the following results.

Table 11 – Outcome Classification group A

		Forecast	
		Illiquid	Liquid
Actual	Illiquid	84,09%	15,91%
	Liquid	14,42%	85,58%

Table 12 – Outcome Classification group B

		Forecast	
		Illiquid	Liquid
Actual	Illiquid	94,29%	5,71%
	Liquid	2,47%	97,53%

Table 13 – Outcome Classification group C

		Forecast	
		Illiquid	Liquid
Actual	Illiquid	97,67%	2,33%
	Liquid	1,25%	98,75%

Since we want to estimate the most reliable model, robustness suggests to pick up the coefficient estimated for the group C, which merge the experience of two portfolio managers.

Coefficients and significance tests (Table 14) appear to be very high, even if not as high as the values obtained with the best discriminant model.

Table 14 – Equation estimate

Variables	Coefficients	E.S.	Significance
Constant	-12,511	1117,100	0,009
V2	1,859	2177,337	0,001
V3	0,373	959,251	0,000
V4	35,457	324,057	0,087
V5	-11,095	435,439	0,020
V6	22,143	2523,745	0,007
V7	-19,462	1047,340	0,015
V8	-1,205	1005,830	0,001
V9	-21,269	280,711	0,060
V10	30,927	595,137	0,041
V11	-12,396	296,357	0,033
V12	-32,794	348,026	0,075

The signs of coefficients are the same of those obtained with the discriminant model, but V5 and V7, positive in the first case and negative in this last one.

5. Conclusions

Our results show how liquidity risk could be measured through inductive methodology applied to static information, not depending upon price time series. In this case, we adopted the experience of two portfolio managers and we tried to estimate a discriminant and a logit equation to replicate the expected output.

Above all, the logit model seems to be the best way to approximate the liquidity risk and the hidden volatility of some bonds, due to little trades on the market.

The factors which help to single out illiquid from liquid bonds are default, listing in exchange markets and, finally, the outstanding.

To conclude our analysis, we highlight the aptitude of our model to reach very high percentage of fitting, especially with the dataset C.

After this study, a benchmark volatility will be applied to every single bond, in order to estimate the liquidity adjusted market risk.

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