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INTEGRATED RISK MANAGEMENT WITH A FILTERED BOOTSTRAP APPROACH

FORTHCOMING ON ECONOMIC NOTES

ABSTRACT

We present a multiperiod risk model to measure portfolio risk that integrates market risk, credit risk and, in a simplified way, liquidity risk. Thus, it overcomes the major limitation currently shared by many risk models, that are unable to give a complete picture of all portfolio risks according to a single, coherent framework. The model is based on the Filtered Bootstrap approach, hence it captures conditional heteroskedasticity, serial correlation and non-normality in the risk factors, that is, most of the features of observed financial time series. Being a simulation risk model it copes in a natural way with derivatives as it allows the full valuation of the probability density function of the contracts. In addition, it is a suitable and flexible way to generate future scenarios on medium term horizons, so this model is particularly appropriate for asset management companies.

KEYWORDS: integrated risk management, market risk, credit risk, liquidity risk, Filtered Bootstrap, risk decomposition, asset management.

JEL CLASSIFICATION: C14, C15, G00.

1. INTRODUCTION

Many risk models have been developed in recent years. A great effort has been spent in modelling market, credit and liquidity risk, but the majority of these models are focused on a single category of risk, so they tend to be appropriate for single business units strongly focused on one type of product.

In particular, credit and market risk have been usually measured separately, not for financial reasons but for practical aspects. From a theoretical point of view, in fact, all the sources of risk should be modelled together in order to consider their interaction on the profit and loss at portfolio level. This would be a very appealing feature, but it comes out to be a difficult task because bonds and equities depend on fairly different risk factors. So it has been natural to first accomplish the task of properly dealing with the risk of the main business area. Some recent research is trying to overcome the limitations associated to the use of separate models, offering alternative solutions for the combined measurement and management of different sources of risk. To the best of our knowledge, all the solutions to this problem are developed in a parametric context. See for example, Iscoe *et al.* (1999), Gil and Polyakov (2003), Medova and Smith (2003).

The literature on market risk is huge, both from the academical and the practitioner's side, and includes a lot of different approaches that include parametric and non-parametric models, closed form and simulation models. For a review and a collection of results on this topic, see, among the others, Jorion (1997), Engel e Gyzicki (1999), Dowd (2002), and www.gloriamundi.org.

Credit risk models are mainly based on a parametric approach, primarily differing from market risk models because they are not directly linked to bonds price time series. This is one of the difficulties in the risk aggregation process. Certainly the most known and successful are CreditMetrics by JP Morgan, KMV (acquired by Moody's) and CreditRisk+ by CSFP (Credit Suisse Financial Products). The first two models belong to the class of structural models and are based on Merton (1974) approach, while the third one is based on an actuarial approach. For a review and a comparison among these models, refer to Crohuy *et al.* (2000), Gordy (1998), Kern *et al.* (1997).

CreditMetrics by JP Morgan (1997) is based on a rating system and on a credit migration matrix. The key aspect of the model is the so called transition matrix which defines for each rating category the probability of being upgraded or downgraded and the probability of default. The credit process is modelled through a stationary Markov chain defined on this transition matrix. In each period a bond can improve or deteriorate its credit quality based on the probability matrix. The future distribution of bonds' returns may be estimated revaluing the bonds in each possible future credit state, using the appropriate yield curve. Finally, to address portfolio risk measurement, the joint distribution of the assets has to be determined. To solve this problem, CreditMetrics models the portfolio by means of a multivariate normal distribution which is calibrated to reflect the historical transition matrix and which is parametrized according to the correlation inferred from the corresponding equity prices.

CreditMetrics is appealing as it deals with credit risk in an intuitive way. The main drawbacks of the model are the following. Credit risk and default risk only depend on the rating category without any contribution from idiosyncratic components, so that each asset in the same rating category share the same risk properties. Furthermore, default probabilities are simple historical estimates based on very few data, especially for non-US countries. Finally, rating categories and default probabilities are seldom updated and are not sensitive to current market conditions.

In KMV, see Sundaram (2001), default probabilities are not a historical measure and do not reflect rating categories. Default rates are continuous variables and are determined through the option pricing approach proposed by Merton (1974). In this approach the capital structure of a firm is decomposed into the equity part and the debt part. The asset value (that is an unobservable variable) is a function of the equity market value and is simulated through a geometric brownian motion. Default happens when the asset value falls under the debt value so that, at least in principle, the firm is not able to repay for its obligations. To measure the likelihood of a default, it is possible to compare the expected value of the distribution of the asset with the debt value. This measure is known as distance to default. The shorter this distance, the higher the expected default frequency.

Finally, in order to deal with the portfolio risk, KMV computes correlations through a multi-factor model that needs a huge amount of data at the level of single firm, industry and country. The KMV model is certainly underpinned by a strong economic basis, but suffers of being data intensive.

CreditRisk+, see Wilde (1997), is not based on a structural modelling of the firm, but on an actuarial estimation of the probability of default. Assuming that this probability is constant over different periods and that the number of defaults is independent over time, the probability distribution of the number of defaults may be modelled as a counting process through the Poisson distribution. The model is entirely described by the expected number of defaults.

Each one of the previous models has its own advantages and disadvantages but all of them share the common feature of not being able to deal with market risk and with optionality in credit products in an integrated way. For example, if you are a balanced portfolio fund manager, you can have a measure of the risk, say Value at Risk (VaR), only for the credit part of the portfolio, but you can not measure the potential loss on the whole of your assets. The best you can do is to sum the expected loss predicted by your credit and market risk models, but in this way you get a biased estimate as you lose the diversification effect. In the same way, if you are a bank that uses risk estimates for capital adequacy reasons, summing risk in an additive way gives you an overestimation of risk. The obvious consequence is to set apart more reserves than you really need, raising the associated opportunity cost.

Other problems may arise with the horizon of the risk analysis. In fact, even if we accept the additive assumption, we could not be able to sum risk forecasts, as different models might predict risk over different time horizons. Traditionally, in the banking environment, market risk is a short term risk, driven by market volatility, which is well known to change very often over time. On the other side, credit and default risks are long term risks. They are defined in terms of the credit worthiness of a company, and they are usually measured over several months. To evaluate integrated market and credit risk, we need a coherent multiperiod model. We have to be able to measure credit risk on short horizons, so it would be desiderable an estimate of the probability of default sensitive to the short-term dynamics of the market. On the other side, we have to model market risk over long term horizons, so we need both conditional and unconditional forecast of the probability distribution associated to market risk.

Things get still harder if we want to consider other aspects, such as liquidity risk. In this case we face many problems. We have to understand what we mean for liquidity risk (an unique definition does not apply). Having relatively few contributions on how to measure it, we have not any consolidated habit in risk management practice and applications. Even if we can deal with these aspects, we have, again, to reconcile all kind of risks together in an unique framework. Among the others, see Cherubini and Della Lunga (2001) and Bangia *et al.* (1999), for an overview on measuring liquidity risk.

To sum up, to be coherent we must be able to do the so called mark to future of the whole portfolio, facing the peculiarities of both market and credit variables over different time horizons. To solve this problem we could, at least in principle, choose a different stochastic process for each risk factor. Using the most realistic approximations we know, we could simulate and reconstruct all the possible outcomes for each factor, obtaining a very precise description of the financial instrument. In principle, we could do this in a parametric context or in a non parametric one. Obviously, this procedure is optimal if we face univariate risk problems, but it becomes more difficult as the number of the processes increases.

To deal with the multivariate nature of the financial markets, we need an unique framework in which the peculiarities of different financial products and their dependencies may be modelled in a robust way. Bootstrap techniques are the solution we propose to solve these problems, as they are a natural way (say non parametric or semi-parametric) to separate the simulation from the choice of stochastic processes and the modelling of the dependence structure with market and credit factors.

The plan of the paper is structured as follows. In section 2 we review the parallel Filtered Bootstrap (FB) approach. In section 3 we illustrate how to integrate market and credit risk in the FB framework, while in section 4 we briefly propose two ways to model liquidity risk in this simulation context. In section 5 we give an example that shows how easy and natural is to analyse risk along different portfolio dimensions using the proposed framework. The final section offers some conclusions.

2. THE FILTERED BOOTSTRAP APPROACH

The risk model relies on the FB technique proposed by Barone-Adesi, Giannopoulos and Vosper (1999), and extensively tested, see for example Zenti and Pallotta (2002). This model considers current market conditions and it is based on the empirical multivariate distribution of risk drivers returns, without imposing any particular probability function. It generates financial scenarios over arbitrary investment horizons using the information content of daily data about the relative or absolute risk that a portfolio might exhibit. This risk model takes into consideration time varying means, volatilities and correlations (as it is applied in a parallel fashion), coping also with serial correlation and leptokurtosis. We give a brief description of the model.

Let us consider, for the sake of simplicity, an equity portfolio. Let us assume that portfolio return depends on N risk drivers. Specifically, we assume that each stock corresponds to a risk driver. The time series of T daily log-returns of the stocks are collected in the $T \times N$ matrix X. Hence, $x_{i,j}$ is the return on the generic day i of the risk driver j.

As well known, the time series of the returns of the risk drivers are usually unsuitable for a naïve application of bootstrapping. If we apply directly the basic bootstrap procedure to the raw returns neither we model conditional volatility, 'missing' volatility clusters, nor we capture any autocorrelation showed by data. From another perspective, the basic bootstrap procedure is based on the assumption that raw returns are identically, independently distributed (i.i.d.). Therefore, if raw returns are not i.i.d., and it is unlikely to be the case, they are unsuitable for bootstrapping and can lead to biased results. The solution is the filtering procedure: the bootstrap is applied to the i.i.d. residuals of a properly chosen model. In our example, a plausible and computationally convenient filter, but not necessarily the only one, is the ARMA(1,1)-GARCH(1,1) model. Therefore, we filter the time series of each stock as follows:

$$x_{i,j} = a + b x_{i-1,j} + c\varepsilon_{i-1,j} + \varepsilon_{i,j}, \qquad \varepsilon_{i,j} \sim \left(0, s_{i,j}\right) \tag{1A}$$

$$s_{i,j}^2 = \alpha + \beta s_{i-1,j}^2 + \gamma \varepsilon_{i-1,j}^2 \qquad j = 1,2,3\ldots, N \tag{1B}$$

where $\varepsilon_{i,j}$ is the generic innovation of the ARMA process and $s_{i,j}^2$ is its variance. Then we bootstrap in a parallel fashion, over the chosen horizon H, the standardized residuals $z_{i,j} = \varepsilon_{i,j} / s_{i,j}$ that are approximately i.i.d. Finally, the paths of i.i.d. standardized residuals we get are used as innovations to simulate the ARMA(1,1)-

GARCH(1,1) process in a high number of scenarios[1]. These scenarios are then used to estimate the multivariate probability density of asset returns and subsequently the ex-ante risk measures over the chosen horizon H. Given that, for each run, the entire path of length H-periods is available, it is possible to estimate the distribution of returns over any horizon $0 < h \leq H$. Thus, the model copes in a proper way with multiple horizons.

It is important to notice that: i) bootstrapping is applied to i.i.d. standardized residuals, so results are unbiased; ii) in general, the residuals $\varepsilon_{i,j}$ are not normal. The parameters of (1A) and (1B) can be estimated using a procedure based on the Pseudo Maximum Likelihood principle.

In general, the filter can be any tractable time series model. For example, for equities, it is often observed that downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. To account for this phenomenon, the filter may correspond to a model that allows for asymmetric shocks to volatility, e.g. Threshold ARCH and Exponential GARCH models. See, among the others, Engle and Ng (1993), Glosten, Jaganathan, and Runkle (1993) and Nelson (1991) on these models. It is possible to select a set of feasible models, using information criteria (e.g. Akaike, Schwarz, Hannan-Quinn) as a guide in the selection of the model for each risk driver. The parallel FB approach, with a properly chosen filter[2], can be used to estimate the probability distribution of any risk factor: equities, currencies, commodities, interest rates.

The estimation of marginal and mean contribution to risk in the context of the FB can be done using the explicit procedure proposed by Hallerbach (2002) for VaR. Traditionally, the calculations of mean and marginal contributions was done numerically, with brute-force, computationally intensive, procedures based on the re-estimation of VaR for a slightly changed portfolio composition (this was generally considered a limitation of any simulation method like the FB approach). Generalising the standard results obtained under normality, Hallerbach derives a distribution-free expression for the marginal and mean contribution of an asset to the diversified portfolio VaR, valid in a simulation context and applicable to non-linear portfolios. The procedure of Hallerbach, which relies on Euler's theorem, can be easily extended to any risk metric which is a homogeneous function of degree one in asset weights. In the Appendix we describe the procedure to derive marginal and mean contributions to risk. Here we give the main results.

Marginal VaR is the partial derivative $\partial VaR_t / \partial V_{i,t}$, where VaR_t is a short notation for the estimate of VaR at time t, over the horizon H, with confidence level cl, while $V_{i,t}$ is the current value in base currency of the

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[1] When using plain Monte Carlo sampling techniques, the number of scenarios is usually set equal to 10,000. However, if the bootstrapping method relies on error reduction techniques, e.g. Quasi Monte Carlo methods, the number of runs can be equal to 1,000 or even less. See Kreinin *et al.* (1998).

[2] Of course, if the time series is i.i.d., there is no need for a filter.

position in asset i. According to the procedure proposed by Hallerbach, and considerin asset i, marginal VaR is given by:

$$MaCVaR_{i,t} = E\left[r_{i,t+H} \middle| r_{p,t+H} = r_{p,t+H}^{cl}\right] \tag{2}$$

where $r_{p,t+H}$ and $r_{i,t+H}$ denote the return on the portfolio and asset i, respectively, estimated through FB over the time horizon H, while $r_{p,t+H}^{cl}$ is the percentile portfolio return over the same horizon that satisfies $\Pr\left[r_{p,t+H} < r_{p,t+H}^{cl}\right] = 1 - cl$, where cl is the desired confidence level (in other terms, $r_{p,t+H}^{cl}$ is the percentage VaR). Hence, the partial derivative $\partial VaR_t / \partial V_{i,t}$, is equal to the conditional expectation $E\left[r_{i,t+H} \middle| r_{p,t+H} = r_{p,t+H}^{cl}\right]$, that is, the expected value of the return on asset i, given that portfolio's return is equal to VaR.

For the mean contribution to VaR we have the following expression:

$$MeCVaR_{i,t} = V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H} = r_{p,t+H}^{cl}\right] = V_{i,t} MaCVaR_{i,t}. \tag{3}$$

In a similar way, it is possible to calculate marginal and mean contributions to portfolio expected shortfall (ES). We define ES with respect to the target return K as $E\left[r_{p,t+H} \middle| r_{p,t+H} < K\right]$, ie it is the conditional expectation of portfolio return, provided it is below the threshold K.

It can be shown that the marginal contribution to portfolio ES is given by the following expression:

$$MaCES_{i,t} = E\left[r_{i,t+H} \middle| r_{p,t+H} < K\right] \tag{4}$$

while for the mean contribution to portfolio ES we have:

$$MeCES_{i,t} = V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H} < K\right] = V_{i,t} MaCES_{i,t}. \tag{5}$$

From the computational point of view, it is clear that the main task is to estimate marginal contributions to risk. Basically, we use the direct sample estimator of the conditional expectations that appears at the second member of (2) and (4). However, when estimating (2), in order to reduce the high estimation error that arises using a single observation[3], we correct the direct sample estimate using the so called adjusted conditional mean estimator. We refer to Hallerbach (2002) for a detailed description of the methodology.

Therefore, accurate estimates of marginal and mean contributions to risk can be obtained in the FB context with great computational efficiency.

3. INTEGRATING MARKET AND CREDIT RISK

Integrating market and credit risk means evaluating the two kind of risk consistently with each other. This has a very practical meaning: one has to employ a model that is able to verify whether the different types of risk offset or enhance each other. The parallel FB approach allows to integrate market risk and credit risk through the simultaneous simulation of credit-risk-free interest rate curves, credit curves (let us call them corporate curves), and the event of default. Intuitively, credit informations come from credit spreads, as they are directly observable variables. By identifying the components of credit risk on each simulated path, we obtain consistent measures for market and credit risk.

The starting point is that interest rates contains several kind of information (see Figure 1):

- the credit-risk-free curve does not contain, by definition, credit information, as it reflects, in a broad sense, macroeconomic conditions (inflation, economic growth, monetary policies and so on);

- the corporate curve (that is, the yield curve estimated from a set of bonds of the same rating and, obviously, currency), or, more precisely, the corporate spread A, reflects information on systemic credit risk conditions;

- the yield of a specific bond, in particular the spread A+B, contains the combined market information on the expected recovery rate in the event of default, the probability of being downgraded as well as the probability of default of a specific bond.

[Insert Figure 1]

More formally, we consider the price of a defaultable bond as a function of its yield to maturity[4], y, which can be thought as the sum of three components:

[3] Note that in a simulation context the expected value of a specific asset's return, given that portolio's return equals VaR, coincides with the single realization, so the sample is made of just one observation.

[4] This is an unnecessary hypothesis: of course, the use of zero coupon curves would be in principle preferible. However, if one has to work with large a universe of bonds, from the pure computational point of view the use of yield curves improves the performance and reduces the size of the database that has to be used to support the simulations, without significantly worsening the quality of the risk analysis.

$$y = y^{crf} + s^{rc} + s^{specific} \tag{6}$$

where:

y^{crf} = credit-risk-free yield corresponding to the same maturity of the bond;

s^{rc} = credit spread, that is, the spread between the corporate yield associated to the same rating and maturity of the bond and the corresponding credit-risk-free yield;

$s^{specific}$ = spread between the bond's yield and the corporate yield curve (that is, $y^{crf} + s^{rc}$);

Hence, if we consider a bond with maturity T, denoting with f_i the cash flow at the generic future time $i \leq T$, the price of the bond at current time t is:

$$B_t(y_t) = B_t\left(y_t^{crf}, s_t^{rc}, s_t^{specfic}\right) = \sum_{j=1}^{T} f_j \cdot \left(1 + y_t^{crf} + s_t^{rc} + s_t^{specfic}\right)^{-j} \tag{7}.$$

As said before, the spread over the credit-risk-free curve, that is $s = s^{rc} + s^{specific}$, contains the market information on credit events, ie downgrade, default, and recovery. We can think that a downgrade is nothing more than an increase in the probability of default. After all, ratings correspond to an ordinal assessment of the likelihood of receiving the promised cash flows at maturity.

Therefore, from the annualised spread s, given an exogenous estimate of the recovery rate, denoted as δ, we can extract an estimate of the annual default probability p^Y using a simplified version of the approach proposed by Ciraolo, Berardi and Trova (2002). Basically, we use a discrete-time version of a reduced-form model, extracting the implied risk-neutral default probabilities. It turns out, see Duffie, Singleton (1999), that an estimate of the annual default probability implied in market credit spreads is given by:

$$p^Y \approx \frac{s}{1-\delta} \quad . \tag{8}$$

If our horizon is less than one year, we must convert the annual probability of default so that it has the proper dimension. We can model the state of the bond as a simple two-states (defaulted/non-defaulted) stationary Markov chain, described by the following daily transition matrix:

$$\Pi^{daily} = \begin{bmatrix} 1-p^{daily} & p^{daily} \\ 0 & 1 \end{bmatrix} \quad (9)$$

where p^{daily} is the daily probability of default. Considering the evolution of the Markov chain, and assuming there are 252 working days per year, we can say that the annual probability, p^Y, is given by:

$$p^Y = 1-\left(1-p^{daily}\right)^{252} . \quad (10)$$

In a similar way one can derive the probability of default on any horizon. From now on, for the sake of simplicity, we will denote the probability of default over the horizon of interest H as p.

Note from (8) that when credit conditions worsen, all the spreads increase, *ceteris paribus*. This implies that if recovery rates do not change[5] significantly, the default probabilities estimated through the formula (8) increase as well for all the corporate bonds analysed. In the rest of this paper, following JPMorgan (1997), we will assume that the recovery rate δ follows a beta distribution, centered on its historical average or on an educated guess from senior portfolio managers. This approach allows us to model in a simple way the uncertainty around the recovery rate.

Armed with an estimate of the default probability and the time series of the yields one can put at work the simulation approach based on the FB framework. In order to avoid negative interest rates and spreads the FB is applied to the first differences of log-yields and spreads. Assuming we are at time t, consider a risk analysis of a quoted corporate bond through parallel FB over a horizon of length H days, using a number of simulated scenarios equal to N. The simulation process works as follows.

- Through FB one gets N, H-days ahead, outcomes for the credit-risk-free yield corresponding to the same maturity of the bond, that is:

$$y^{crf}_{t+H,i}, \quad i = 1,2,\ldots,N .$$

- At the same time, as the simulation is conducted in a parallel way on all the risk drivers, one generates also N, H-days ahead, scenarios for the spread between the corporate and the credit-risk-free yield, that is:

$$s^{rc}_{t+H,i}, \quad i = 1,2,\ldots,N .$$

[5] Recovery rates are relatively stable as they depend mainly on the industry: it is reasonable to assume, for example, that IT firms have lower recovery rates than automotive firms, as the latter can sell more phisical assets in the case of default.

- From the spread s and a vector of beta variates representing δ, using (8), one gets N outcomes for the probability of default p:

$$p_i \ , \quad i = 1,2,\ldots,N \ .$$

- From (7), using the current value of the credit-risk-free yield, the corporate spread and the specific spread, that is, $y_t^{crf}, s_t^{rc}, s_t^{specfic}$, one gets the current bond price B_t.

- The risk due to movements of the credit-risk-free curve can be estimated from the probability distribution of returns calculated as follows:

$$R_{t+H,i}^{crf} = \frac{\sum_{j>t+H} f_j \cdot \left(1 + y_{t+H,i}^{crf} + s_t^{rc} + s_t^{specfic}\right)^{-j} + cashflow_{t,t+H}}{B_t} - 1 \ , \quad i = 1,2,\ldots,N \tag{11}$$

assuming for simplicity that the cashflow $cashflow_{t,t+H}$ is invested in a current account without remuneration. Basically, the idea is to analyse the behaviour of the bond when the only changing risk factor is the credit-risk-curve.

- In a similar way, the risk due to movements of the credit-spread can be estimated from the probability distribution of returns calculated as follows:

$$R_{t+H,i}^{rc} = \frac{\sum_{j>t+H} f_j \cdot \left(1 + y_t^{crf} + s_{t+H,i}^{rc} + s_t^{specfic}\right)^{-j} + cashflow_{t,t+H}}{B_t} - 1 \ , \quad i = 1,2,\ldots,N \ . \tag{12}$$

- The risk due to joint changes in the level of credit spread and credit-risk-free rate is then estimated from the probability distribution of the following returns:

$$R_{t+H,i}^{crf+rc} = \frac{\sum_{j>t+H} f_j \cdot \left(1 + y_{t+H,i}^{crf} + s_{t+H,i}^{rc} + s_t^{specfic}\right)^{-j} + cashflow_{t,t+H}}{B_t} - 1 \quad i = 1,2,\ldots,N \ . \tag{13}$$

- The risk associated to default can be calculated from the following distribution of returns:

$$R_{t+H,i}^{default} = \begin{cases} \delta_i - 1 & with\ probability \quad p_i \\ 0 & with\ probability \quad 1-p_i \end{cases}, \quad i = 1,2,...,N \tag{14}$$

which is the simple simulation of a binomial event.

- Finally, the overall level of risk of the bond can be calculated from the following distribution of returns, that takes into account all the sources of risk considered by the model:

$$R_{t+H,i}^{total} = \begin{cases} \delta_i - 1 & with\ probability \quad p_i \\ R_{t+H,i}^{crf+rc} & with\ probability \quad 1-p_i \end{cases} \quad i = 1,2,...,N\ . \tag{15}$$

Technically, to identify the scenarios corresponding to the event of default, we proceed as it follows. For each defaultable bond, we sample from an uniform distribution a random vector with as many rows as the number of scenarios. The next step is to compare, in every scenario, each random realization with the probability scaled for the proper time horizon as in (9). If the uniform random number (that may be considered a probability, being defined on the domain [0,1]) is greater than the probability of default, the bond does not default in that scenario, and viceversa.

In this way, if we have N scenarios and NB bonds, the event of default is identified at the horizon H, through the following:

$$\begin{cases} U_i^j > p_i^{j,H} \Rightarrow the\ bond\ does\ not\ default & i = 1,2,...,N;\ j = 1,2,...NB \\ U_i^j \leq p_i^{j,H} \Rightarrow the\ bond\ defaults & i = 1,2,...,N;\ j = 1,2,...NB \end{cases} \tag{16}$$

where U_i^j is the (i, j) realization of the uniform variate, and $p_i^{j,H}$ is the corresponding estimated default probability on the relevant time horizon.

After locating in the simulation all the scenarios in which defaults happen, we can attribute to those scenarios the expected return in case of default, that is the recovery rate minus one, as in (14-15). Obviously, if a bond defaults in a scenario at a given time horizon, that bond is still considered defaulted for the entire path corresponding to that scenario.

This procedure has some aspects that have to be noticed. First, as the generation of uniform numbers is independent across different bonds, two assets with the same default probability may experience insolvency in two different scenarios, even if, at the level of the entire distribution, they will have the same number of paths defaulted. Second, we simulate defaults and recovery rates in an independent way. This means that when there is insolvency, the expected return of the bond is centered on an expected recovery rate, which has a variability that is only connected to the uncertainty around this estimate. This choice is due to the fact that according to this approach the moment in which the firm defaults is an idiosyncratic component, while the amount that the firm is able to repay to creditors depends on a variable, which may be centered on different values, but that is characterised by the same uncertainty across assets.

Finally, note that we do not simulate directly the specific spread $s^{specific}$, as its time series can be problematic; for example, it can be very short if the bond has been recently issued. The information content of $s^{specific}$ is taken into account during the simulation of the default process, as $s^{specific}$ is involved in (8), which is used to determine the default probability.

Having estimated the probability distribution of portfolio returns over the desired horizon through (11), (12), (13), (14), (15), a variety of summary statistics and distribution descriptors can be calculated and used as risk indicators. From the computational point of view, this is the easiest part of the work, as one has just to pick up some metrics.

This procedure is applied in a parallel fashion to each bond in the portfolio under examination. This allows the calculation of portfolio risk metrics. Note that at portfolio level the comovements among risk drivers are captured in a natural way by the parallel bootstrap mechanism. The only exception concerns the default process: although it is possible to introduce some dependence in this process, we do not model explicitly default correlation. Anyway, we capture some degrees of dependence, as the estimated default probabilities tend to increase when credit spreads widen (and credit conditions worsen). Empirical literature on estimating correlation of default risk is rather scarce, mainly due to data limitations. Regarding estimation of correlations, see Frey *et al.* (2001).

4. LIQUIDITY RISK

There are many definition of liquidity risk. See Cherubini and Della Lunga (2001) and Bangia *et al.* (1999) for a survey on liquidity risk. Going back to the first principles, a particularly appealing, economic meaningful definition of liquidity risk is discussed in Kyle (1985). He suggests a tripartite definition of liquidity risk, that is: i) 'tightness', the cost of turning around an asset in short time, ii) 'depth', the size of the order flow needed to change prices of a give amount, iii) 'resilience', the recovery speed of prices after uninformative shocks. Ideally, we should incorporate these ideas in our analysis, but this is easier said than done. Hence we adopt a somewhat limited definition of liquidity risk. In particular, we identify liquidity risk with the risk that bid-ask spreads widen: the bigger the spread, the less liquid the market. If one accepts this plausible, although incomplete, notion of illiquidity, liquidity risk can be modelled in a rather natural way in the FB framework. This can be done in a non-parametric way or using a simple parametrization form.

4.1. NON-PARAMETRIC APPROACH

Given the fact that bid-ask quotes are often observable, to account for the impact of the bid-ask spread, one can directly bootstrap the bid-ask spreads.

If the bootstrap is applied directly to bid-ask spreads, for each of the N runs, the result is a path of H bid-ask spreads $\phi_{i,j}$, $j = 1,2,...,H$, $i = 1,2,...,N$, where the last bid-ask spread, $\phi_{H,i}$ is the one of interest. For example, in order to keep into account liquidity risk for a long position in a particular bond, the return calculated on mid-prices has to be reduced by the quantity:

$$l_i = -\tfrac{1}{2}\frac{\phi_{H,i}}{B_t},\ i = 1,2,...,N. \tag{17}$$

This approach has a number of shortcomings. First, it is important to keep in mind that in many instances, the quoted spread differs from the effective spread, that is, the spread between the actual prices of a sell order and a buy order: for example, many transactions occur at prices within the quoted bid-ask spread. Secondly, this limitation can be in practice overwhelmed by the one that arises when there is a serious lack of market data on bid ask-spreads. In this case, it can be impossible to put at work the approach outlined above, but one can still incorporate liquidity risk in the FB framework using the methodology described below.

4.2. PARAMETRIC APPROACH

Roll (1984) proposes a simple model that allows to infer the (effective) bid-ask spread directly from the time series of market prices. Roll shows that the bid-ask spread ϕ is a function of $\sigma_{\Delta P}$, the standard deviation of the first differences of the market prices:

$$\phi = \sigma_{\Delta P}\sqrt{2}. \tag{18}$$

If the horizon of the risk analysis is H, the FB approach allows to simulate the distribution of returns, and hence of prices, for a sequence of horizons, namely $1,2,...,H$. In other words, one gets N paths of length H for the predicted prices. From the first differences of these prices, using (18), it is possible to calculate the distributions of bid-ask spreads for the horizon H, ie $\phi_{H,j}$, $j = 1,2,...,N$. Given that distribution, the final step is to apply (17).

5. AN EXAMPLE

We give an example of portfolio analysis using the approach outlined before. Our goal is to show how easy and natural is to analyse risk along different dimensions using the same methodological framework. In order to give a very tractable example, we analyse a simple portfolio exposed to both market and credit risk (see Table 1). We perform the risk analysis using a horizon equal to 1 month. We show how risk can be decomposed both along the asset dimension (ie marginal and component risk) and along the risk dimension, observing the way risk is distributed among market, credit and liquidity risk. See Table 2. In this example we do not split credit risk in its main drivers (credit spreads' movements and default occurrence) but this of course not only possible, but also natural.

[Insert Table 1]

[Insert Table 2]

Then we consider a specific bond, EL PASO CORPORATION EP 7 1/8 05/06/09, see Table 3. Among the corporate bonds this is the one that contributes the most to risk, as it exhibits the greatest component risk. We analyse the marginal distributions respectively related to i) movements of the credit-risk-free curve; ii) bid-ask spreads' movements; iii) changes in the level of credit spreads and default's occurence; iv) joint movements of all the sources of risk considered. See Figure 2. If one looks at the distribution of bond's returns due to credit risk drivers, it is apparent the presence of some tail events, associated to the defaults.

[Insert Table 3]

[Insert Figure 2]

6. CONCLUSIONS

We propose an approach to the simultaneous assessment of market, credit and liquidity risk based on the parallel Filtered Bootstrap, which relies on an empirical multivariate probability distribution inferred from financial data in a semi-parametric way. Thus, the model captures most of the features observed in financial time series, such as conditional heteroskedasticity, autocorrelation and leptkurtosis. As bootstrap is applied in a parallel way to all the risk factors, their comovements are captured in a semi-parametric way, without any restrictive hypothesis about the correlation structure, that in general is not linear and it is variable over time.

Being a simulation risk model based on a single, coherent framework, it copes naturally with multiple time horizons, using all sources of information relevant to market, credit risk and liquidity risk for large investment sets of financial instruments. The parallel FB approach allows to integrate market risk and credit risk through the simultaneous simulation of credit-risk-free interest rate curves, corporate curves, and the event of default. Information on systemic and specific credit risk, default included, are extracted from credit spreads, as they are directly observable variables, using some mild assumptions on the distribution of recovery rates. If market data on bid-ask spreads are available, liquidity risk can be easily integrated in the proposed approach, as the bootstrapping procedure can be applied directly to bid-ask spreads. If this kind of data is unavailable, liquidity risk can be inferred from the simulated time series of market prices, using the model proposed by Roll (1984).

Like any simulation model it allows the full valuation of the probability density function of derivatives.

These features characterize a suitable and flexible way to generate future scenarios both on short and medium term horizons. Therefore, this model is particularly appropriate for asset management companies with a broad investment universe, usually managing several balanced portfolios, with multiple horizons to keep under control.

APPENDIX

We outline the procedure to get a decomposition of any risk metric, which is a homogeneous function of degree one in asset positions (or weights), in a generic simulation framework. We refer to Hallerbach (2002) for a complete formal proof.

Let us focus on a portfolio of N assets with current value $V_{p,t}$ and consider a generic risk metric $Risk$ that admits a decomposition in marginal and mean contributions, denoted $MeCRisk$ and $MaCRisk$, respectively.

The mean contribution of asset i to the overall level of portfolio $Risk$ is defined according to the following expression:

$$Risk = \sum_{i=1}^{N} MeCRisk_i \,. \tag{A1}$$

The marginal contribution of asset i to portfolio $Risk$ is defined as:

$$MaCRisk_i = \frac{\partial Risk}{\partial V_i} \tag{A2}$$

where V_i is the value of the position in asset i, expressed in base currency.

Given the current values of the positions in the N assets, $V_{i,t}$, the change in portfolio value over the chosen horizon H is:

$$\Delta V_{p,t+H} = V_{p,t} r_{p,t+H} = \sum_i V_{i,t} r_{i,t+H} \tag{A3}$$

where $r_{p,t+H}$ and $r_{i,t+H}$ denote the return on the portfolio and asset i, respectively, in our case estimated through FB. Provided that all the marginal return distributions have finite first moments, we can write (A3) in a different form, applying the basic definition of conditional expectation:

$$\Delta V_{p,t+H} = E\left[\Delta V_{p,t+H} \middle| r_{p,t+H}\right] = \sum_i V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H}\right]. \tag{A4}$$

As $\Delta V_{p,t+H}$ is a continuous and homogeneous function of degree one in asset positions, we can apply Euler's theorem, that is:

$$\Delta V_{p,t+H} = \sum_i V_{i,t} \frac{\partial \Delta V_{p,t+H}}{\partial V_{i,t}}. \tag{A5}$$

Substituting (A5) in (A4) gives us the following fundamental expression for the change in portfolio value over the chosen time horizon H:

$$\Delta V_{p,t+H} = \sum_i V_{i,t} E\left[\frac{\partial \Delta V_{p,t+H}}{\partial V_{i,t}} \middle| r_{p,t+H}\right]. \tag{A6}$$

We define the percentage portfolio VaR over the horizon H with confidence level cl as the percentile portfolio return $r^{cl}_{p,t+H}$ that satisfies $\Pr\left[r_{p,t+H} < r^{cl}_{p,t+H}\right] = 1 - cl$. Hence portfolio VaR in base currency, ie not in percentage, is simply $\Delta V^{cl}_{p,t+H} = V_t r^{cl}_{p,t+H}$ (note that VaR is not expressed in terms of losses[6]).

Now, considering equation (A6) and (A4), if we condition on $r_{p,t+H} = r^{cl}_{p,t+H}$, that is on percentage VaR, we can write the following expression for portfolio VaR:

$$\Delta V^{cl}_{p,t+H} = \sum_i V_{i,t} E\left[\frac{\partial \Delta V_{p,t+H}}{\partial V_{i,t}} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right] = \sum_i V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right] \tag{A7}$$

From (A7) it turns out that:

[6] This allows us to calculate VaR for financial products that do not admit losses. For example, the domain of the probability distribution of returns associated to capital guarantee products at maturity exhibits a lower bound equal to zero. Hence in this case VaR is a positive real number.

$$E\left[\frac{\partial \Delta V_{p,t+H}}{\partial V_{i,t}} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right] = E\left[r_{i,t+H} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right] \tag{A8}$$

and from (A2) it then follows that the marginal contribution to VaR is:

$$MaCVaR_{i,t} = E\left[r_{i,t+H} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right]. \tag{A9}$$

From equation (A1), (A7) and (A9) we get the mean contribution to VaR:

$$MeCVaR_{i,t} = V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H} = r^{cl}_{p,t+H}\right] = V_{i,t} MaCVaR_{i,t}. \tag{A10}$$

Similarly, it is possible to calculate marginal and mean contributions to portfolio expected shortfall (ES). We define ES with respect to the target return K as $E\left[r_{p,t+H} \middle| r_{p,t+H} < K\right]$, ie it is the conditional expectation of portfolio return, provided portfolio return are below the threshold K.

Following the same logic, it can be shown that the marginal contribution to portfolio ES is given by the following expression:

$$MaCES_{i,t} = E\left[r_{i,t+H} \middle| r_{p,t+H} < K\right] \tag{A11}$$

while for the mean contribution to portfolio ES we have:

$$MeCES_{i,t} = V_{i,t} E\left[r_{i,t+H} \middle| r_{p,t+H} < K\right] = V_{i,t} MaCES_{i,t}. \tag{A12}$$

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Caption of Figure 1:

The information content of interest rates as considered in our model.

Table 1

Description	Security type	Rating Bloomberg Comp.	W_{PTF}
TELECOM ITALIA SPA	Equity	-	15%
ALCATEL SA ALAFP 4 3/8 02/09	Corp. Bond	B+	20%
EL PASO CORPORATION EP 7 1/8 05/06/09	Corp. Bond	B-	25%
GOLDMAN SACHS GROUP INC GS 4 1/4 08/04/10	Corp. Bond	A+	40%
Total			**100%**

In order to give a tractable example, we analyse a simple portfolio exposed to both market and credit risk. Table 1 shows the main portfolio attributes: security description, security type (bond/equity), rating, asset's weight.

Table 2

Description	ES = E[R_{PTF} | R_{PTF}<0]		VaR(5%)		Decomposition of VaR(5%)		
	Marginal	Component	Marginal	Component	Liquidity Component	Market Component	Credit Component
TELECOM ITALIA SPA	-3.91%	-0.59%	-7.76%	-1.16%	-0.11%	-0.60%	0.00%
ALCATEL SA ALAFP 4 3/8 02/09	-0.97%	-0.19%	-1.25%	-0.25%	-0.02%	-0.05%	-0.47%
EL PASO CORPORATION EP 7 1/8 05/06/09	-1.72%	-0.43%	-1.20%	-0.30%	-0.03%	-0.05%	-0.58%
GOLDMAN SACHS GROUP INC GS 4 1/4 08/04/10	0.07%	0.03%	0.14%	0.06%	-0.03%	-0.31%	-0.22%
Total	**-**	**-1.18%**	**-**	**-1.66%**	**-0.18%**	**-1.00%**	**-1.27%**

Picture of risk of the examined portfolio over a horizon H = 1 month.

For each asset, from left to right we show:

- the holding's description;
- marginal and component expected shortfall (ES), where ES is calculated with respect to the target return K (we set $K = 0$), ie, denoting portfolio return over the chosen horizon as $r_{p,t+H}$, ES is defined as $E\left[r_{p,t+H} \middle| r_{p,t+H} < K\right]$;
- marginal and component Value at Risk (VaR) with probability level 5%, where VaR is simply the quantile of the returns' distribution;
- the VaR decomposition by type of risk, ie liquidity (using the parametric approach), market, and credit risk.

It is evident some diversification among risks, as portfolio VaR is –1.66%, while the sum of liquidity, market and credit VaR is –2.46%.

Table 3

EL PASO CORPORATION EP 7 1/8 05/06/09
ISIN: XS0147412083
Currency: EURO
Rating: B- (Bloomberg Composite)
Current* yield to maturity: 9.25%
OAS*: 5.52%
Modified Duration*: 4.05
** As of 30/12/2003, source Bloomberg*

Main characteristics of one of the corporate bonds held in the examined portfolio.

Caption of Figure 2

Estimates of the marginal probability distribution functions (mpdf) for the bond EL PASO CORPORATION EP 7 1/8 05/06/09. Clockwise and top-down, we show the mpdf respectively related to i) movements of the credit-risk-free curve; ii) bid-ask spreads' movements; iii) changes in the level of credit spreads and default's occurence; iv) joint movements of all the sources of risk considered.